General Considerations of Electrical Measuring Instruments 12

12.1 INTRODUCTION

Measurements are the basic means of acquiring knowledge about the parameters and variables involved in the operation of a physical system. Measurement generally involves using an instrument as a physical means of determining a quantity or variable. An instrument or a measuring instrument is, therefore, defined as a *device for determining* the value or magnitude of a quantity or variable. The electrical measuring insturment, as its name implies, is based on electrical principles for its measurement function.

These days a number of measuring instruments, both analog as well as digital ones, are available for the measurement of electrical quantities like voltage, current, power energy, frequency, power factor, *etc*. The instruments considered in this book are analog devices in which the output or display is a continuous-time signal and bears a fixed relationship to the input.

Analog instruments may be divided into three groups:

(*a*) Electromechanical instruments;

(*b*) Electronic instruments which are often constructed by the addition of electronic circuits to electromechanical indicators thus increasing their sensitivity and input impedances; and

(*c*) Graphical instruments which are electromechanical and electronic instruments having a modified display arrangement so that a graphical trace, that is, a display of instantaneous values against time is obtained.

This chapter presents general concepts related to the working principles and construction and certain features common to many electrical indicating instruments particularly of electromechanical types. Important definitions relevant to instruments will be discussed first.

12.2 DEFINITIONS OF IMPORTANT TERMS

Measurement work employs a number of terms which are defined below:

Measurand: The quantity or variable being measured is called measurand or measurement variable.

Accuracy: It is defined in terms of the closeness with which an instrument reading approaches the true or expected (desired) value of the variable being measured.

Precision: It is measure of the consistency of reproducibility (repeatability) of the measurement (*i.e.*, the successive reading do not differ). For a given fixed value of an input variable, precision is a measure of the degree to which successive measurement differ from one another.

Sensitivity: It is defined by the change in the output or response of the instrument for a unit change of input or measured variable.

Resolution: Resolution is the smallest change in a measured variable (or measurand) to which the instrument will respond.

True or Expected Value: The true or expected value of a quantity to be measured may be defined as the average of an infinite number of measured values when the average deviation due to the various contributing factors tends to zero. It also refers to a value of the quanity under consideration that would be obtained by a method (known as exemplar method) agreed upon by experts. In other words, it is the most probable value that calculations indicate and one should expect to measure.

Note that the value of the unknown obtained by making use of primary standards and measuring instruments is considered to be its ture value.

Error: It is the deviation of the measured (or indicated) value from the true (or expected) value of a quantity. In other words, error is the difference between the measured value and the true value of the unknown quantity. It is also called *absolute error are maximum possible error*. Then error of measurements is given by

$$\in_A = A_m - A_t \qquad \text{...(12.1)}$$

where A_m = measured value of the quantity

A_t = true value of the quantity

Absolute error, $\in_0$, is the limit of error in measurement. In other words, $\in_A$ must not be higher than $\in_0$.

Thus, $$|\in_0| = \max |A_m - A_t| \qquad \text{...(12.2)}$$

Note that the absolute error does not give any information about accuracy. For example, an error of (–1) volt in measurement of 1000 volt is negligible, but the same error in measurement of 10 volts is never acceptable. Thus, error is expressed in terms of another term called the *relative error* which is the ratio of absolute error of the ture value of the quantity being measured. Therefore, the relative error, $\in_R$ is given by

$$\in_R = \frac{\text{Absolute error}}{\text{True value}} = \frac{\in_0}{A_t}$$

$$= (A_m - A_t)/ A_t \qquad \text{...(12.3)}$$

The percentage relative error % $\in_R = \in_R \times 100$.

Also, from Eqn. (12.3), we have

$$(1 + \in_r)\, A_t = A_m$$

or $$A_t = \frac{A_m}{1+[\in_R]} \qquad \text{...(12.4)}$$

If the absolute error $\in_A$ is sufficiently small, then Eqn. (12.1) shows that

$$\in_A = A_m - A_t \approx 0$$

or $$A_t \cong A_m \qquad \text{...(12.5)}$$

That is, A_m may be substituted for A_t in Eqn. (12.3) for practical purpose. Now in view Eqn. (12.3) becomes

$$\in_R = \frac{\in_0}{A_m} \qquad ...(12.6)$$

Correction: The difference between the true value and the measured value of the sought quantity is defined as the reading correction or simply *correction*. That is, correction is negative or error. Thus,

$$\delta C = -\in_A \qquad ...(12.7)$$

$$= A_t - A_m \qquad ...(12.8)$$

or

$$A_t = \text{expected value} = A_m + \delta C \qquad ...(12.9)$$

Therefore addition of correction in measured value gives the true (or accurate or expected) value.

Bandwidth: The bandwidth of an instrument relates to the maximum range of frequency over which it is suitable for use. It is normally quoted in terms of 3 *dB* (*dB* = decibel) point. For an amplifier, it is the range of frequencies between which the gain or amplitude ratio is constant to within 3 *dB* (this corresponds 30% reduction in gain).

Significant Figures: An indication of the precision of the measurement is obtained from the number of significant figures in which it is expressed.

Significant figures convey actual information regarding the magnitude and the measurement precision of a quantity. The more is the significant figures, the greater will be the precision of measurement. For example, if a resistor is specified as having a resistance of 105 Ω and 105.3 Ω, than in 105 Ω there are three significant figures whereas in 105.3 Ω there are four. The later with more significant figures, expresses a measurement of greater precision than the former.

12.3 CLASSIFICATION OF MEASURING INSTRUMENTS

Electrical measuring instruments may be classified into two groups:

(*a*) Absolute (or primary) instruments.

(*b*) Secondary instruments.

12.3A Absolute Instruments

- These instruments give the value of the electrical quantity in terms of absolute quantities (or some constants) of the instruments and their deflections.
- In this type of instruments no calibration or comparison with other instruments is necessary.
- They are generally not used in laboratories and are seldom used in practice by electricians and engineers. They are mostly used as means of standard measurements and are maintained lay national laboratories and similar institutions.
- Some of the examples of absolute instruments are:
  * Tangent galvanometer
  * Raleigh current balance
  * Absolute electrometer.

12.3B Secondary Instruments

- They are direct reading instruments. The quantity to be measured by these instruments can be determined from the deflection of the instruments.
- They are often calibrated by comparing them with either some absolute instruments or with those which have already been calibrated.

- The deflections obtained with secondary instruments will be meaningless untill it is not calibrated.
- These instruments are used in general for all laboratory purposes.
- Some of the very widely used secondary instruments are: ammeters, voltmeter, wattmeter, energy meter (watt-hour meter), ampere-hour meters etc.

Classification of Secondary Instruments

(*a*) Classification based on the various effects of electric current (or voltage) upon which their operation depend. They are:

- **Magnetic effect:** Used in ammeters, voltmeters, watt-meters, integrating meters etc.
- **Heating effect:** Used in ammeters and voltmeters.
- **Chemical effect:** Used in dc ampere hour meters.
- **Electrostatic effect:** Used in voltmeters.
- **Electromagnetic induction effect:** Used in ac ammeters, voltmeters, watt meters and integrating meters.

Generally the magnetic effect and the electromagnetic induction effect are utilized for the construction of the commercial instruments. Some of the instruments are also named based on the above effect such as electrostatic voltmeter, induction instruments, etc.

(*b*) Classification based on the Nature of their Operations

We have the following instruments.

- **Indicating instruments:** Indicating instruments indicate, generally the quantity to be measured by means of a pointer which moves on a scale. Examples are ammeter, voltmeter, wattmeter etc.
- **Recording instruments:** These instruments record continuously the variation of any electrical quantity with respect to time. In principle, these are indicating instruments but so arranged that a permanent continuous record of the indication is made on a chart or dial. The recording is generally made by a pen on a graph paper which is rotated on a dice or drum at a uniform speed. The amount of the quantity at any time (instant) may be read from the traced chart. Any variation in the quantity with time is recorded by these instruments. Any electrical quantity like current, voltage, power etc., (which may be measured lay the indicating instruments) may be arranged to be recorded by a suitable recording mechanism.
- **Integrating instruments:** These instruments record the consumption of the total quantity of electricity, energy etc., during a particular period of time. That is, these instruments totalize events over a specified period of time. No indication of the rate or variation or the amount at a particular instant are available from them. Some widely used integrating instruments are: Ampere-hour meter: kilowatthour (kWh) meter, kilovolt-ampere-hour (kVARh) meter.

(*c*) Classification based on the Kind of Current that can be Measurand.

Under this heading, we have:

- Direct current (dc) instruments
- Alternating current (ac) instruments
- Both direct current and alternating current instruments (dc/ac instruments).

(*d*) Classification based on the method used.

Under this category, we have:

- **Direct measuring instruments:** These instruments converts the energy of the measured quantity directly into energy that actuates the instrument and the value of the unknown quantity is measured or displayed or recorded directly. These instruments are most widely used in engineering practice because they are simple and inexpensive. Also, time involved in the measurement is shortest. Examples are Ammeter, Voltmeter, Watt meter etc.
- **Comparison instruments:** These instruments measure the unknown quantity by comparison with a standard. Examples are dc and ac bridges and potentiometers. They are used when a higher accuracy of measurements is desired.

(*e*) Classification based on the Accuracy Class of Instruments.

Groups of error in the measured quantity for instruments of various class of accuracy are listed below:

Class of accuracy	0.2	0.5	1.0	1.5	2.5	5
Limit of error %	± 0.2	± 0.5	± 1.0	± 1.5	± 2.5	± 5

12.4 FEATURES COMMON TO ALL INDICATING INSTRUMENTS

In this section we will discuss certain features which are common to all electrical measuring instruments. We will first consider various torques acting on its moving system. In an indicating instrument, it is essential that the moving system is acted upon by three distinct torque (or forces) for satisfactory working. There torques are:

1. A deflecting or operating torque, T_d
2. A controlling torque, T_c
3. A dampling torque, T_v.

12.4A Deflecting (Or the Operating) Torque

The deflecting torque, causes the moving system of the instrument to move from its zero position. It may be produced by utilizing any one of the effects of current or voltage in the instrument such as magnetic effect, electromagnetic induction effect, heating effect, electrostatic effect etc. The actual method of producing a deflecting torque depends upon the type of the instruments.

The deflecting torque has to supply the following torque-components presents in an instrument.

(*a*) The torque required to overcome the torque due to the inertia of the moving system, $J\,(d^2\theta/dt^2)$, where J is the moment of inertia and θ is the movement (rotation in radians).

(*b*) The torque required to overcome the controlling torque, T_c $(\equiv k_c\theta)$.

(*c*) The torque required to overcome the damping torque, $T_v \left(= k_v \dfrac{d\theta}{dt}\right)$, where k_v is damping torque constant.

(*d*) The torque requirred to overcome the frictional (coulomb) torque. This component is minimized by appropriate design considerations.

12.4B Controlling Torque

The controlling torque developed in an instrument has two functions:

(*a*) It limits the movement of the moving system and ensures that the magnitude of the deflections always remains the same for a given value of the quantity to be measured.

(*b*) It brings back the moving system to its zero position where the quantity being measured is removed or made zero.

The controlling torque is dependent on the magnitude of deflection produced. The moving system is deflected from zero to such a position that the controlling torque at that deflected position is equal to the deflecting torque. The controlling torque increases in magnitude with the deflection till it balances the deflecting torque. That is, for a steady deflection,

Controlling torque, T_c = Deflection or operating torque, T_d ...(12.10)

The controlling torque is entered in all commercial instruments by any one of the following three ways.

- By means of one or two coiled springs. The corresponding insturment is termed *spring controlled instruments* (mostly used system).
- By the action of gravity due to suitably placed weights on the moving system. Such instruments are known as *gravity controlled instruments*.
- By means of a permanent magnet (magnetic control system).

Spring control is now almost universal in indicating instruments. Gravity control is employed in a few cases, notably in special laboratory types, and magnetic control is applied to some galvanometers and certain moving iron instruments (the polarized form). We will discuss the first two methods of obtaining the controlling torque in a measuring instrument as given below.

Spring Control

Figure 12.1(*a*) shows a spindle free to turn between two pivots. The moving system is attached to the spindle. Two phosphor-bronze hair springs *A* and *B* wound in opposite directions are also shown whose inner ends are attached to the spindle. The outer end of spring *A* is connected to a leaver which is pivoted the adjustment of which gives zero setting. However, the outer end of *B* is fixed.

When the pointer is deflected one spring unwinds itself while the other is twisted. This twist in the spring produces restoring (controlling) torque, which is proportional to the angle of deflection of the moving systems.

Let *E* be the young-modulus for the material of the spring and θ (radians) be the deflection of the moving system to which one end of the spring is attached. Then, the controlling torque developed in the spiral spring is given by

$$T_c = \frac{Ebt^2}{12l}\theta \qquad ...(12.11)$$

or

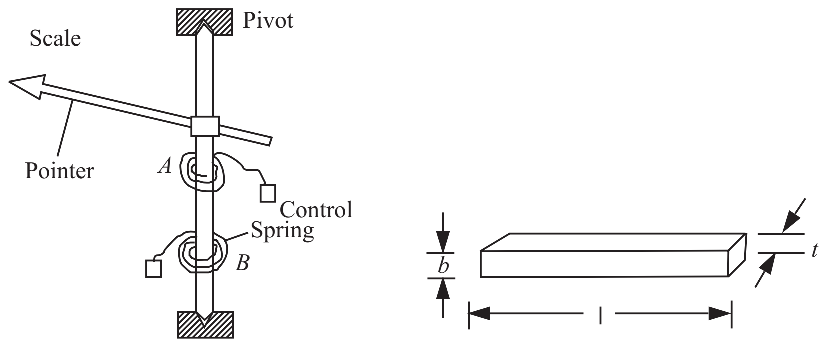
$$T_C = k_s\,\theta \qquad ...(12.12)$$

where l = Total length of spring strip (*m*)

b = depth of the strip (*m*)

t = thickness of the strip (*m*)

k_s = spring constant



(*a*) Pivoted-moving system with spring control

(*b*) Spring in strip form

Fig. 12.1

The controlling spring must meet the following requirements:

- The stress developed in the spring must be well below the elastic limit of the spring material at the maximum deflection of the moving system. This is essential to avoid fatigue and to preserve stability over a long period. For this, we must have

$$\frac{l}{r} = \frac{E\theta}{2S_{\max}} \qquad ...(12.13)$$

  where $S_{\max}$ = maximum stress which must not exceed. For a full scale deflection $\theta = 90°$, the ratio l/t is about 3000 in a good instruments.

- It springs are used as leads of current to the instrument, their cross-sectional area must be sufficient to carry the current without overheading them failing which the consistency will be impaired. The spring material should also have the following properties:

  * It should have low resistance

  * The temperature coefficient should also be low.

- The springs must be of non-magnetic material.

In a permanent magnet moving coil type instrument the deflecting torque is proportional to the current passing through them. Thus the operating torque, T_d, is directly proportional to the current,

$$T_d = KI \qquad ...(12.14)$$

Then for spring control instrument, the controlling torque, T_C, is

$$T_c = K_s\theta \qquad ...(12.15)$$

The pointer comes to rest when the deflecting torque (T_d) and the controlling or restoring torque (T_c) are equal, *i.e.*, T_d is equal and opposite to T_c.

At equilibrium, $\quad T_d = T_c$

Therefore, $\quad KI = K_s\theta$

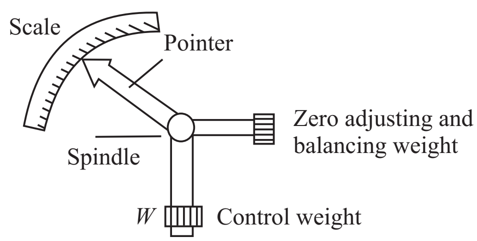
$$\therefore \qquad I = \frac{K_S\theta}{K} \qquad ...(12.16)$$

This equation shows that the current is directly proportional to the deflection and since Eqn. (12.16) is a linear relation, the scale with spring controlled instrument for deflecting torque given by Eqn. (12.14) will be uniform throughout the scale.

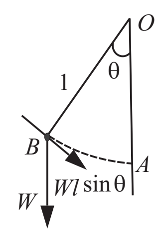
Gravity Control

In gravity controlled instruments, as shown in Fig. 12.2 (*a*) a small adjustable weight is attached to the spindle of the moving system such that the deflecting torque produced by the instrument has to act against the action of gravity. Thus a controlling torque is obtained. This weight is called the *control weight*. Another adjustable weight is also attached is the moving system for zero adjustment and balancing purpose. This weight is called *Balance weight*.

When the control weight is in vertical position as shown in Fig. 12.2 (*a*), the controlling torque is zero and hence the pointer must read zero. However, if the deflecting torque lifts the controlling weight from position A to B as shown in Fig.12.2 (*b*) such that the spindle rotates by an angle θ, then due to gravity a restoring (or controlling) torque is exterted on the moving system.



(*a*) Gravity controlled moving system at zero deflection



(*b*) Moving system rotated by θ radian

Fig. 12.2

The controlling (or restoring) torque, T_c, is given by

$$T_c = Wl \sin\theta = k_g \sin\theta \quad \text{...(12.17)}$$

where W is the control weight; l is the distance of the control weight from the axis of rotation of the moving system; and k_g is the gravity constant.

Equation (12.18) shows the controlling torque can be varied quite simply by adjustment of the position of the control weight upon the arm which carries it.

Again, if the deflecting torque is directly proportional to the current, I *i.e.*,

$$T_d = kI \quad \text{...(12.18)}$$

We have at the equilibrium position

$$T_d = T_c$$

or
$$kI = k_g \sin\theta$$

or
$$I = \frac{k_g}{k} \sin\theta \quad (12.19)$$

This relation shows that current I is proportional to sin θ and not θ. Hence in gravity controlled instruments the scale is not uniform. It is cramped for the lower readings, instead of being uniformly divided, for the deflecting torque assumed to be directly proportional to the quantity being measured.

Advantanges of Gravity Control

1. It is cheap and not affected by temperature variations.
2. It does not deteriorate with time.
3. It is not subject to fatigue.

Disadvantages of Gravity Control

1. Since the controlling torque is proportional to the sine of the angle of deflection, the scale is not uniformly divided but cramped at its lower end.
2. It is not suitable for use in portable instruments (in which spring control is always preferred).
3. Gravity control instruments must be used in vertical position so that the control weight may operate and also must be leveled otherwise they will give zero error.

In view of these reasons, gravity control is not used for indicating instruments in general and portable instruments in particular.

12.4C Damping Torque

We have already seen that the moving system of the instrument will tend to move under the action of the deflecting torque. But on account of the control torque, it will try to occupy a position of rest when the two torques are equal and opposite. However, due to inertia of the moving system, the pointer will not come to rest immediately but oscillate about its final deflected position as shown in Fig. 12.3 and takes appreciable time to come to steady state.

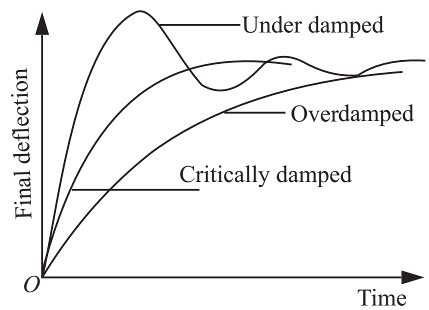


Fig. 12.3 *Dynamic response of a measuring instrument*

To overcome this difficulty a damping torque is to be developed by using a damping device attached to the moving system. The damping torque is proportional to the speed of rotation of the moving system, that is

$$T_v = k_v \frac{d\theta}{dt}$$

where k_v = damping torque constant

$\frac{d\theta}{dt}$ = speed of rotation of the moving system

Depending upon the degree of damping introduced in the moving system, the instrument may have any one of the following conditions as depicted in Fig.12.3.

1. **Under damped condition:** The response is oscillatory
2. **Over damped condition:** The response is sluggish and it rises very slowly from its zero position to final position.
3. **Critically damped condition:** When the response settles quickly without any oscillation, the system is said to be critically damped.

In practice, the best response is slightly obtained when the damping is below the critical value *i.e.*, the instrument is slightly under damped.

The damping torque is produced by the following methods:

Air Friction Damping

In this type of damping a light vane or vanes having considerable area is attached to the moving system to develop a frictional force opposing the motion by reason of the air they displace. Two methods of damping by air friction are depicted in Fig.12.4.

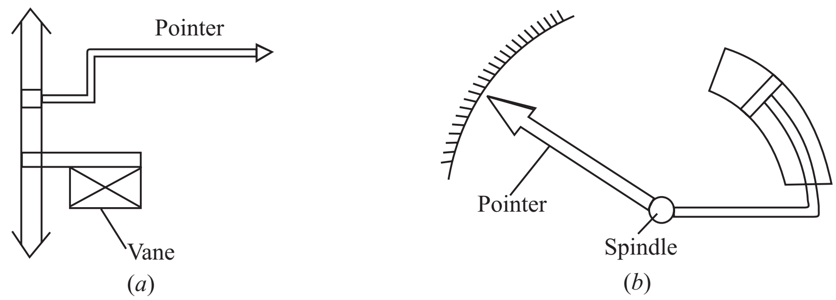


Fig. 12.4 *Air-friction damping*

- The arrangement shown in Fig. 12.4(*a*) consists of a light aluminium vane which moves in a quadrant (sector) shaped air chamber. The chamber also carries a cover plate at the top. The vane is mounted on the spindle of the moving system. The aluminium vane should not touch the air-chamber walls otherwise a serious error in the deflection of the instrument will be introduced. Now, with the motion, the vane displaces air and thereby a damping force is created on the vane that produces a torque (damping) on the spindle. When the movement is quicker the damping force is greater; when the spindle is at rest, the damping force is zero.
- The arrangement of Fig.12.4 (*b*) consists of a light aluminium piston which is attached to the moving system. This piston moves in a fixed chamber which is closed at one end. Either circular or rectangular chamber may be used. The clearance (or gap) between the piston and chamber walls should be uniform thorughout and as small as possible. When the piston moves rapidly into the chamber the air in the closed space is compressed and the pressure of air thus developed opposes the motion of the piston and thereby the whole moving system. If the piston is moving out of the chamber, rapidly, the pressure in the closed space falls and the pressure on the open side of the piston is greater than that on the opposite side. Motion is thus again opposed. With this damping system care must be taken to ensure that the arm carrying the piston should not touch the sides of the chamber during its movement. The friction which otherwise would occur may introduce a serious error in the deflection.

The air friction damping is very simple and cheap. But care must be taken to ensure that the piston is not bent or twisted. This method is used in moving iron and hot wire instruments.

Fluid Friction Damping

- This form is damping is similar to air friction damping. The action is the same as in the air friction damping. Mineral oil is used in place of air and as the viscosity of oil is greater, the damping force is also much greater. The vane attached to the spindle is arranged to move in the damping oil.
- It is rarely used in commercial type instruments.
- The oil used must fulfill the following requirements.
  * It should not evaporate quickly
  * It should not have any corrosive effect on metals.
  * Its viscosity should not change appreciably with temperature.
  * It should be good insulator.

Two arrangements of fluid damping are shown in Fig. 12.5.

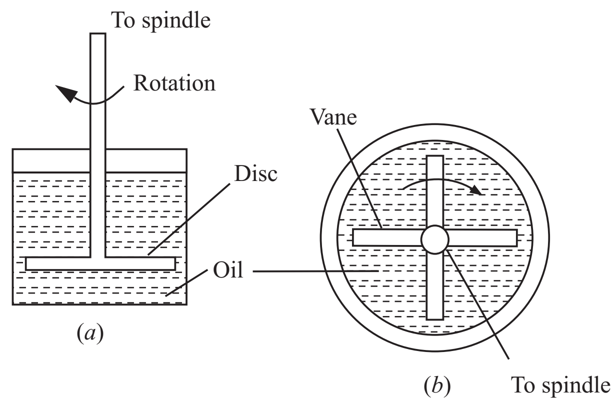


Fig. 12.5 *Fluid friction damping devices*

(*a*) In Fig. 12.5(*a*) a disc attached to the moving system is immersed in the fluid (damping oil). When the moving system moves the disc moves in oil and a frictional drag is produced. For minimizing the surface tension affect, the suspension stem of the disc should be cylindrical and of small diameter.

(*b*) In the arrangement of Fig. 12.5(*b*) a number of vanes are attached to the spindle. These vanes are submerged in oil and moves in a vertical plane. This arrangement provides greater damping torque.

Advantages of Fluid Friction Damping

1. The oil used for damping can also be used for insulation purpose in some forms of instruments which are submerged in oil.
2. The clearance between the vanes and oil chamber is not as critical as with the air friction clamping system.
3. This method is suitable for use with instruments such as electrostatic type where the movement is suspended rather than pivoted.
4. Due to the up thrust of oil, the loads on bearings or suspension system is reduced thereby the reducing the frictional errors.

Disadvantages of Fluid Friction Damping

1. The instruments with this type of damping must be kept always in a vertical position.
2. It is difficult to keep the instrument clean due to leakage of oil.
3. It is not suitable for portable instruments.

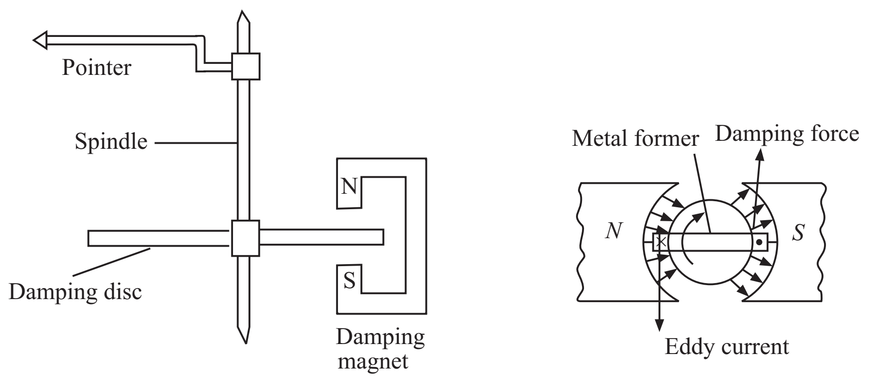
The fluid friction damping can be used for laboratory type electrostatic instruments.

Eddy Current Damping

Eddy current damping is the most efficient form of damping. The essential components in this type of damping are a permanent magnet; and a light conducting disc usually of alumninum.

When a sheet of conducting material moves in a magnetic field so as to cut through lines of force, eddy currents are set up in it and a force exists between these currents and the magnetic field, which is always in the direction opposing the motion. This force is proportional to the magnitude of the current, and to the strength of field. The former is proportional to the velocity of movement of the conductor, and thus, if the magnetic field is constant, the damping force is proportional to the velocity of the moving system and is zero when there is no movement of the system.

Figure 12.6 shows two methods of applying this method of damping. In Fig. 12.6(*a*) a thin disc of conducting, but non-magnetic material-usually copper of aluminium is mounted on the spindle which carries the pointer of the instrument. When the spindle rotates, the edge of the disc cuts through the lines of force in the gap of a permanent magnet, and eddy currents, with consequent damping, are produced. An arrangement similar to this is often used in hotwire instruments.



(*a*) Eddy current damping (*b*) Electromagnetic damping

Fig. 12.6

Figure 12.6(*b*) shows the essential parts of a permanent-magnet, moving coil, instrument. The coil is wound on a light metal former in which eddy currents are induced when the coil moves in the permanent-magnet field. The directions of the eddy-current which in turn produce the damping torque due to the motion of the coil (clockwise) are as shown in Fig.12.6(*b*) and this will produce damping forces as indicated in the figure.

Electromagnetic Damping

- The movement of a coil in a magnetic field produces a current in the coil which interacts with the magnetic field to produce a torque. This torque opposes the movement of the coil and slows the response.
- The magnitude of the current and hence the damping torque is dependent upon the resistance of the circuit which the instrument is connected.
- This damping method is used in galvanometers.

12.5 CONSTRUCTIONAL DETAILS OF INDICATING INSTRUMENTS

Special cares are essential in the design and construction of some components (parts) of the indicating instruments. The components considered in the section are:

- Supports for moving system
- Permanent magnet
- Pointer and Scale
- Case.

Supports for Moving System

The main requirement to be fulfilled by a supporting system is that the friction should be as minimum as possible. The two commonly used methods for supporting the moving system of an instrument are:

1. By pivoting and 2. By thread suspension.

Most instruments use the supports of first kind. In the case, the ends of the spindle are conical and are made of hardened steel. The ends fits into jeweled bearings of conical shape made from aluminium oxide. The contract area at the pivots should be as small as practicable. However a very small area of contact leads to a very high bearing stress.

The thread suspension systems have limited applications in commercial instruments due to following reasons:

- The instrument must be leveled and its axis must be vertical.
- It must be protected against mechanical shocks.

The method is advantageous where the operating torques are small compared with weight of the moving system since the friction is completely avoided. Phosphor bronze strips are commonly used for suspension.

Permanent Magnets

Instruments having permanent magnets as their main component, it is essential to ensure that the strength of the permanent magnets be constant over a considerable time period. Materials used for construction of such magnets are:

- Alloys of cobalt, chromium and steel
- Almico (or Alcomax)
- Alloys of iron, nickel and aluminium.

Pointers and Scales

Pointers and scales of instruments may be classified together into two groups:

- Instruments used for reading at considerable distance.
- Instruments used for precisions work at shoot range.

It is essential that the pointer must be light and must have small inertia constant so as to reduce the load in the bearing of the moving system and to avoid high degree of damping. Its outline must be bold with sharp pointer in the first type. We often use aluminium strip on tube for the pointer. The scale of an instrument of first category is mostly printed on the enameled surface of a metal plate, or on paper or card-board cemented rigidly to a metal backing plate.

For the precision (work) in reading, a strip of mirror is mounted in an opening in the scale beneath the pointer. The reading is taken by removing the parallax error between the position and its image in the mirror.

Cases:

- The main function of a case is to make the instrument dust and moisture proof.
- Steel cases are used to provide magnetic screening for insturment which are affected by external magnetic fields.
- To reduce the error due to hysteresis and eddy currents effects, the moving system of instruments should be mounted in a position far away from the metal case.

12.5A Balancing

For a perfect mechanical balance of the moving systems, the centre of gravity should always lie on the axis of rotation. When this is ensured then the deflection of a spring-controlled instruments will be independent of its position, and the wear on the bearings will be uniform.

In the gravity control system the balancing is done both by control and balancing weight. The distance and weight of control and balancing weight are decided on the basic of their effects upon the weight and inertia of the moving system.

In one of the methods of obtaining fine balancing in spring control is that the pointer axis is prolonged on the other side of the pivot and small metal is fixed on it. The metal contains small screws by means of which balancing is obtained. This is illustrated in Fig. 12.7.

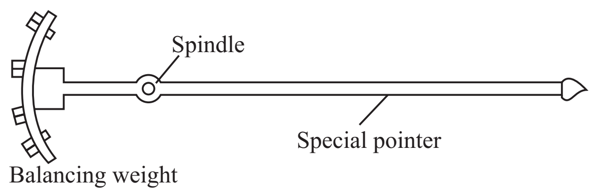


Fig. 12.7 *Balancing of moving system by using a special pointer*

12.5B Torque/Weight Ratio

In order to reduce the load on the bearings and to reduce the frictional torque (proportional to the pressure on the bearing surface), the weight of the moving should be made as small as possible.

The ratio of the deflecting torque (in Nm, when it acts at a radius of 1 metre) to produce full scale deflection to the weight of the moving system in kg should always be more than 0.1. This ratio is influenced by whether the axis of moving system is vertical or horizontal.

12.6 DYNAMIC RESPONSE OF A MEASURE INSTRUMENT

It has already been mentioned earlier that when a electric current flows along a conductor, the conductor becomes surrounded by a magnetic field. This property is utillized in electro-mechanical instrument to obtain the motion of the spindle of the moving system in the following ways:

(*a*) By interaction of the magnetic field around a coil with a permanent magnet;

(*b*) Between ferromagnetic vanes in the coils magnetic field; or

(*c*) Through the interaction of the magnetic fields produced by a number of coils.

Constraining these forces to form a tuning moment, a deflecting or operating torque to is produced which is a function of the current in the instrument's coil, the geometry and the type of the coil system. This torque is given by

$$T_d = Gf\ (i),\ \text{Nm} \qquad ...(12.20)$$

where G = instrument constant; and $f(t)$ is some function of current.

To obtain a stable display, it is necessary to equate the deflection torque with an opposing or control torque. The opposing torques are:

(*a*) Inertial torque, $T_j = jd^2\ \theta/dt^2$; where J is the moments of inertia of the moving system.

(*b*) Damping torque, $T_v = Dd\theta/dt$; D is the damping constant

(*c*) Control torque $T_c = C\theta$; C is the spring constant (if the instrument is assumed to be have a spring-control system) and it is the rotation (in radius) of the moving system.

Equating the deflecting torque to the total opposing torques, the equation of motion for a pointer (or indicating) instrument becomes:

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + C\theta = Gf\,(i)$$

The steady-state solution is given by

$$C\theta_{ss} = Gf\,(i)$$

whereas the dynamic or transient solution will have the form

$$\theta_{tr} = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

where A and B are arbitrary constants.

$$\lambda_1 = -\frac{D}{2j} + \sqrt{\frac{D}{4j^2} - \frac{C}{J}},\ \lambda_2 = -\frac{D}{2J} - \sqrt{\frac{D^2}{4J^2} - \frac{C}{J}}$$

For a particular instrument C and J are fixed in magnitude during manufacturer but D (the amount of damping) may be varied. This results in three possible modes of transient response.

(*a*) When $D^2/J^2 > C/J \rightarrow$ for which the roots λ_1 and λ_2 are real and unequal, the response is known as the over damped response curve (*a*) in Fig. 12.3.

(*b*) When $D^2/J^2 > C/J \rightarrow$ for which the roots are real and equal, D has a value termed the critical value, the response is known as the critical damped response; curve (*b*) in Fig. 12.3.

(*c*) When $D^2/J^2 > C/J \rightarrow$ for which the roots are complex-conjugate, the system is under damped, curve (*c*) in Fig. 12.3. The frequency of decaying oscillations being.

Figure 12.3 illustrates that the magnitude of the damping applied to a movement has an important effect on the dynamic performance of an instrument. In general, as mentioned earlier also, pointer (or indication) instruments are usually operated (provided) with slightly less than the pointer changes (moving) rapidly from one position to another with the minimum change of sticking.

■ **Example 12.1**. An instrument spring, constructed of phosphorbronze strip has the following dimensions:

Length of strip = 375 mm, Thickness of strip = 0.1 mm, and Width of strip = 0.45 mm.

Of Young's modulus of elasticity of the spring material is 1.15×10^{10} kg per m^2. Calculate the controlling torque extended by the spring when it turned through an angle of 60°.

Solution. The controlling torque in an spring controlled instrument is given by

$$T_C = Ebt^3/12l\theta \qquad \ldots(i)$$

where E = Young's modulus of the spring material.

$$= 1.15 \times 10^{10}\ \text{kg/m}^2$$

$$b = \text{width of the strip} = 0.45\ \text{mm}$$

$$t = \text{thickness of strip} = 0.1\ \text{mm}$$

$$l = \text{length of the strip} = 375\ \text{mm}$$

$$\theta = \text{rotation of the spindle (radius)} = 60° = \pi/3$$

Substituting these values in (*i*), we have

$$T_C = (1.15 \times 10^{10}) \times (0.45 \times 10^{-3})^3 \times \pi/3/12 \times 375 \times 10^{-3}$$

$$= 1.204 \times 10^{-6}\ \text{kg m.}$$

■ **Example 12.2.** A weight of 5 gram is used as the controlling weight in a gravity controlled instrument. Find its distance from the spindle, if the deflecting torque corresponding to a deflection of 60° is 1.15×10^{-4} kg m.

Solution. In gravity controlled instrument at the equilibrium deflection position of the moving system, the deflecting torque T_d equals the controlling torque, T_C. That is

$$T_d = T_c = Wl \sin \theta$$

Then, $$l = \frac{T_c}{W \sin \theta} = 1.15 \times 10^{-4} / 5 \times 10^{-3} \times \sin 60° = 2.26 \text{ am.}$$

■ **Example 12.3.** In a certain recording instrument, the electromagnetic torque (deflecting torque), corresponding to a full scale deflection of 60° is 1.1×10^{-4} kgm. The control is exerted through two phosphor-bronze spiral springs: The allowable maximum stress in the phosphor-bronze is 6×10^6 kg/m^2 and the Young's modulus of elasticity E of the spring material is 12×10^{10} kg/m^2. Calculate the suitable dimension for springs. Assume the width of the spring strip to be mm.

Solution: As there are two control springs, the controlling torque produced by each spring is equal to half of the deflecting torque. That is, the controlling torque of each spring is

$$T_c = 1.1 \times 10^{-4}/2 = 0.55 \times 10^{-4} \text{ kgm}$$

Also, $$T_c = \frac{Ebt^3}{12l}\theta$$

or $$\frac{t^3}{l} = \frac{12T_c}{Eb\theta}, \theta \text{ is in radians} = \frac{\pi}{3}$$

$$t^3/l = 0.526 \times 10^{-10} \quad ...(i)$$

The length to thickness ratio is given by

$$\frac{l}{t} = \frac{E\theta}{2S_{\max}}$$

or $$l = 1047.62t \quad ...(ii)$$

Then, Eqn. (*i*) gives

$$t^3 = 0.526 \times 10^{-10} \times 1$$

$$t = 0.2347 \text{ mm}$$

and $$l = 1047.62 \times 0.2347 \times 10^{-3}$$

$$= 24.59 \text{ cm.}$$

■ **Example 12.4.** The deflecting torque of an ammeter is directly proportional to the current thorugh it of a current of 10 A deflects the pointer by 90°, find the value of current for a deflection of 60° when the insturment is (*a*) spring controlled and (*b*) gravity controlled.

Solution. The deflecting torque

$$T_D = kl \quad ...(i)$$

(*a*) For spring controlled instrument

$$T_C = k_S \Theta \quad ...(ii)$$

At equilibrium position

$$T_D = T_C$$

$$kI = k_S \Theta$$

or $$I = \left(\frac{k_S}{k}\right)\theta = k_1\,\theta$$

or $$I_2 = \left(\frac{\theta_2}{\theta_1}\right) I_1 = 6.667 \text{ A.}$$

(*b*) For gravity controlled instrument.

$$T_C = Wl \sin\Theta = k_g \sin\Theta$$

At equilibrium position

$$I = \left(\frac{k_g}{k}\right)\sin\Theta = k_2 \sin\Theta$$

Then $$\frac{I_2}{I_1} = \frac{\sin\theta_2}{\sin\theta_1} \text{ gives}$$

$$I_2 = \frac{\sin 60°}{\sin 90°} \times 10 = 8.66 \text{ A.}$$

Example 12.5. The deflecting torque of an ammeter varies as square of the current through it. It the deflection corresponding to a current of 12 A is 90°, find the deflection for a current of 6 A when the instrument is (*a*) spring controlled and (*b*) gravity controlled.

Solution. The operating or deflecting torque

$$T_d = kI^2$$

(*a*) For spring controlled instrument

$$T_d = T_c = k_S\theta$$

$$kI^2 = k_S\theta$$

or $$I^2 = \left(\frac{k_S}{k}\right)\theta = k_1\theta$$

Then $$\left(\frac{I_2}{I_1}\right)^2 = \frac{\theta_2}{\theta_1}$$

or $$\theta_2 = 22.5°.$$

(*b*) For gravity controlled instrument,

$$T_O = T_C = k_G \sin\theta = KI^2$$

Then $$\frac{\sin\theta_2}{\sin\theta_1} = \frac{I_2^2}{I_1^2}$$

$\therefore$ $$\theta_2 = \sin^{-1}(1/4) = 14.48°.$$

■ **Example 12.6.** The coil of a moving coil galvanometer has 300 turns and is suspended in a uniform magnetic field of 0.1 Wb/m^2 by a phosphor bronze strip of which the torsion constant is 2×10^{-7} Nm/rad. The coil is 2 cm wide and 2.5 cm high with a moment of inertia of 1.5×10^{-7} kgm^2. If the galvanometer resistance is 200 Ω, find the value of resistance which when connected across the galvanometer terminals will give cirtical damping. Assume that the damping is entirely by electromagnetic.

Solution. Number of turns, $N = 300$

Magnetic field, $B = 0.1$ Wb/m^2

Area of coil, $A = 0.02 \times 0.025 = 0.0005$ m^2

Displacement constant, G is

$$G = NBA = 300 \times 0.1 \times 0.0005 = 0.15 \text{ rad/A}$$

Inertia constant, $J = 1.5 \times 10^{-7}$ kgm^2

Restoring constant, $C = 2 \times 10^{-7}$ Nm/rad

For critical damping, damping constant

$$D = \sqrt{4JC} = \sqrt{4} \times (1.5 \times 10^{-7}) \times (2 \times 10^{-7}) = 3.46 \times 10^{-7}$$

Since damping is entirely due to electormagnetic action, resistance required (R_C) for critical damping is given by

$$R_C = \frac{G^2}{D} = \frac{(0.105)^2}{3.46 \times 10^{-7}} = 650 \text{ Ω}$$

Additional resistance required for critical damping condition

$$= R_C - \text{galvanometer resistance}$$
$$= 650 - 200 = 450 \text{ ohms.}$$

SUMMARY

In this chapter we have studied classification of instruments and certain features common to all indicating instruments. Important consideration related to the construction and designs have also been mentioned. A brief account has also be given for the dynamic response of indicating instruments. In the next chapter we shall discuss the working principles of various electrical instruments with their features and typical applications.

FURTHER READING

1. Golding, E.W., "*Electrical Measurements and Measuring Instruments*", Sir Isaac Pitman, London, 1960.
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3. Davis, Barthlomew, "*Electrical Measurements and Instrumentation*", Allyn and Bacon, Inc., Boston, 1965.
4. Melville, B. Stout "*Basic Electrical Measurements*", *2nd Ed.*, *Chapter 2, Englewood Clitt's NJ*. Prentice-Hall, Inc., 1960.
5. Prasad, Rajendra, "*Electrical Measurements and Measuring Instruments*", Khanna Publishers, Delhi, 1979.

PROBLEMS

12.1 Differentiate between absolute and secondary instruments and explain which one of them has wider practical application.

12.2 Distinguish among indicating recording and integrating instruments, Give at least one example of each type.

12.3 What effect of an electric current is used is secondary instruments? Based on the above effects, give a stable of commercial type of instruments indicating their important features.

12.4 Explain the necessity of deflecting controlling and damping torques in an indicating instrument. How the instrument would behave if any one of the torques is absent?

12.5 Compare gravity control with spring control. State their relative merits and demerits.

12.6 What are three differnent types of damping provided in electrical measureing instruments which one of them is considered must efficient and why? Can it be provided with all types of instruments?

12.7 What is understand when it is said that the moving system of a measuring instrument is over, critically and under damped? What is the state of damping in a dead beat instrument?

12.8 Develop a dynamic model of a dc galvanometer carrying a current I and also find its dynamic response if the back emf is given by

$$e = G\frac{d\theta}{dt}$$

where G = the galvanometer constant (Nm/A) and θ is the deflection at any time t in rad

Assume the resistance of the galvanometer circuit to be R and other parameters (constants) as given below:

J = moment of inetia of the moving system about the axis of rotation, kgm^2

D = daming constant (Nm/(rad/second))

C = elastic constant of the suspension (Nm/radian).

12.9 A phosphor-bronze spring has the following dimensions:

Length of strip = 400 mm

Thickness of strip = 0.075 mm

Width of strip = 0.5 mm

The Young's modulus of phosphor-bronze is 12×10^9 kg/m^2. Estimate the torque exerted by the spring when it is turned through 90°. [**Ans.** 0.83×10^{-6} kgm]

12.10 In a gravity controlled instrument the deflecting torque corresponding to a deflection of 90° is 1.2×10^{-4} kgm. Find the controlling distance of 2 cm from the axis of moving system. [**Ans.** 0.6×10^{-2} kg]

12.11 The deflecting torque in a spring-controlled instruments is 1.2×10^{-4} kgm for a deflection of 90°. The control torque is exerted through a phosphor-bronze spring whose Young's modulus is 1.15×10^{10} kg/m^2 of the allowable maximum stress is 5.5×10^6 kg/m^2 and width of spring strip is 0.75 mm, And the thickness and length of suitable dimensions for the spring-strip. [**Ans.** 0.685 m, 0.4175 mm]

12.12 If the deflecting torque of an instrument is directly proportional to the current to be measured and the maximum current produces a deflection of 90°, compare the deflections in a spring controlled instrument with a similar instrument having gravity control for a current equal to half the maximum value also comment upon the two types of control methods.[**Ans.** 45°, 30°]

12.13 The deflecting torque of an ammeter varies as the square of the current flowing through it. If a current of 10 A produces a deflection of 90°, what deflection would occur for a current of 5 A when the instrument is (*a*) spring controlled and (*b*) gravity controlled? [**Ans.** (*a*) 22.5°, (*b*) 14.5°]

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